North Dakota
Standards and Benchmarks

Content Standards

Mathematics

1999

North Dakota Department of Public Instruction
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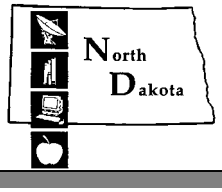

The publication of this work is funded wholly or in part by grant CFDA Number 84.168R from the National Eisenhower Mathematics and Science Programs, U.S. Department of Education. Its contents do not necessarily reflect the views or policies of the Department, or any other agency of the U.S. Government.

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Components of the Document

Content Standards – general statements that describe what students should know and the skills they should have in a specific content area.

Benchmarks – statements of knowledge and skill that define a standard at a given developmental level.

Examples of Specific Knowledge – facts, vocabulary, principles, generalizations, relationships, concepts, step-by-step procedures, strategies, or processes that describe the specific information or skills that students should acquire to meet a standard.

Examples of Activities – instructional activities that students could do to acquire the knowledge and skills described in the standard and benchmarks.

** Indicates the word or phrase is defined in the Glossary*

Introduction

The North Dakota Mathematics Content Standards Project has produced this document to reflect the needs of North Dakota Schools, based on the expertise and experience of practicing teachers and collegiate mathematics educators in the state. The importance of mathematical literacy as a central educational outcome for the children and youth of North Dakota is widely recognized. Achieving goals requires thoughtful planning. The National Council of Teachers of Mathematics (NCTM) produced a series of Standards documents (1989, 1991, 1995, 1998) designed to help teachers, administrators and parents recognize high quality mathematics education. While the NCTM Standards documents provide useful information in various settings, states and local districts must make their own interpretations of local needs for high quality mathematics education.

Criteria Shaped the Standards Document
Several criteria guided the writing of these mathematics standards. While recognizing the reality of existing mathematics education practice in North Dakota, the document also provides a realistic vision of what mathematics education can be in the state. The document provides content, rather than curriculum standards, recognizing the importance of aligning curricula with needs in a local context. The standards document is primarily intended to serve as a guide for curricular planning at the local district level; it also should be a useful, compact reference for classroom teachers.

Document Structure
This document is organized around a core of fundamental mathematics standards. Benchmarks for each standard were written for three levels, reflecting desired achievement in mathematics during early elementary (K-4), middle school (5-8), and high school years (9-12).

Except for some additional material listed for college-intending, senior high school students (marked with an asterisk), the benchmarks reflect mathematical knowledge that is important for all students in our schools by the target grade levels (grades 4, 8, and 12). We have not intended that each benchmark necessarily be covered during each year in the group.

We have included sections labeled “Examples of Specific Knowledge…” and “Examples of Activities…” with each standard to help the reader interpret the standard and benchmarks. Neither the listing of specific knowledge nor the sample of activities is intended to be comprehensive, but each is included to clarify the benchmarks.

As the team looked for the core areas of mathematics that should be included in the standards, several themes emerged that ran through all of the content areas. These common themes are included as content goals and permeate all the standards.

The glossary includes terms that may be necessary to clarify the standards and benchmarks for the reader. The glossary is not intended to be a comprehensive mathematics dictionary.

Quality Mathematics Instruction Is More Than Content
The writing team believes strongly that mathematics education develops continuously over each student’s educational career. Further, good mathematics education requires much more than high quality content. Mathematics instruction should reflect what both educational research and empirical knowledge of best practice have to say about the teaching and learning of mathematics. For example, the literature emphasizes that mathematics instruction at all levels should include hands-on experiences, use of manipulatives, student inquiry, and integrated and regular use of appropriate technologies, including calculators and computers. Students need to

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do more than just listen to teacher expositions and work textbook problems; they should spend time on a regular basis generating, discussing and writing about mathematical ideas.

Goals for Mathematics Programs

Mathematical achievement of the United States must improve to ensure our nation's continued leadership in a global technological society. Today's students will be tomorrow's workforce with social and economic challenges that will require different mathematical skills than those of the industrial age.

To meet the changing needs of mathematics education for North Dakota students and to prepare mathematically literate citizens, the following six goals should be supported in the mathematics program of the schools:

- Students will become mathematical problem solvers.
- Students will be able to reason mathematically.
- Students will be confident in their mathematical abilities.
- Students will be able to communicate mathematically.
- Students will be able to make mathematical connections.
- Students will be able to use appropriate technology.

The chart on the following page describes each goal as it pertains to the student and mathematics curriculum. Making the six goals an integral part of each standard and benchmark will prepare students to meet the challenges of the twenty-first century.

The problems and challenges facing today's youth demand that all students experience a rich mathematics curriculum which provides access to the kind of knowledge that will help them solve problems and respond to any challenge. This document is intended to help focus on the essential elements of a world-class program in mathematics for all North Dakota students.

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Mathematics Goals	Importance of the Goal	Student Who Has Attained Goal Will:	Components Needed In Mathematics Curriculum
Students will become mathematical problem solvers.	• develops flexibility, perseverance, and strategies • organizes and interprets information • forms abstractions and generalizations	• find ways to solve problems when no routine path is apparent	• frequent and regular opportunities to solve non-routine problems, including those students pose themselves
Students will be able to reason mathematically.	• fundamental to "doing" mathematics • involves using patterns, generalizations, and abstractions	• gather data, make conjectures, assemble evidence, and build arguments to support or refute conjectures	• opportunities to acquire mathematics facts and skills • opportunities to express and interpret math ideas and relationships • situations that require inductive and deductive reasoning skills
Students will be confident in their mathematical abilities.	• promotes positive attitude toward mathematics and recognition that mathematics is a common human activity • encourages trust in own mathematical thinking	• use mathematics to make sense of new problem situations	• numerous and varied types of mathematical experiences • successful learning experience
Students will be able to communicate mathematically.	• prepares one to use mathematics effectively and efficiently • allows progress to higher levels of mathematics • helps to clarify, refine, and consolidate thinking	• translate information from real world into mathematical language using signs, symbols, graphs, and terms of mathematics • present mathematical ideas in written, visual, and oral formats	• opportunities to read, write, and discuss ideas in problem situations • opportunities to think and talk about mathematics by asking questions and working with others
Students will be able to make mathematical connections.	• makes mathematics meaningful and useful • develops flexibility in career options	• make links among mathematical ideas and to other disciplines • portray mathematics as an integrated whole that permeates inside and outside of school	• situations that require application of mathematics facts, skills, concepts, and processes to problems encountered in life
Students will be able to use appropriate technology.	• expands traditional problem solving techniques • extends application of mathematics to real-world • improves comprehension and understanding	• know how to use appropriate technology to solve meaningful mathematical problems • understand and appreciate the power and limitations of technology	• many opportunities to use technology (electronic devices as well as hands-on manipulatives) to explore, perform, and organize mathematical strategies and processes

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North Dakota Mathematics Content Standards

Standard 1: NUMBER AND OPERATION
Students understand and use basic and advanced concepts of number and number systems.

Standard 2: GEOMETRY AND SPATIAL SENSE
Students understand and apply geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations.

Standard 3: DATA ANALYSIS, STATISTICS AND PROBABILITY
Students use data collection and analysis techniques, statistical methods, and probability to solve problems.

Standard 4: MEASUREMENT
Students use concepts and tools of measurement to describe and quantify the world.

Standard 5: ALGEBRA, FUNCTIONS AND PATTERNS
Students use algebraic concepts, functions, patterns, and relationships to solve problems.

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Summary of Grades K – 4 Benchmarks

Standard 1: NUMBER AND OPERATION

Students understand and use basic and advanced concepts of number and number systems.

4.1.1 Construct and interpret number meanings through real-world experiences.
4.1.2 Understand the characteristics and properties of our numeration system.
4.1.3 Understand how arithmetic operations are related to one another in addition, subtraction, multiplication, and division.
4.1.4 Rename, order, and compare numbers.
4.1.5 Know and use basic facts and computational algorithms for whole numbers, fractions and decimals.
4.1.6 Use estimation strategies in working with quantities, measurement, computation, and problem solving.
4.1.7 Understand and communicate strategies to solve a wide variety of problems.

Standard 2: GEOMETRY AND SPATIAL SENSE

Students understand and apply geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations.

4.2.1 Know the characteristics of two- and three-dimensional shapes.
4.2.2 Understand how two- and three-dimensional shapes can be changed by combining and dividing.
4.2.3 Understand that geometry is found within and outside mathematics.

Standard 3: DATA ANALYSIS, STATISTICS AND PROBABILITY

Students use data collection and analysis techniques, statistical methods, and probability to solve problems.

4.3.1 Collect, organize and display data.
4.3.2 Formulate and solve problems that involve data.
4.3.3 Draw conclusions based on probability.
4.3.4 Use technology and materials as tools to display data.

Standard 4: MEASUREMENT

Students use concepts and tools of measurement to describe and quantify the world.

4.4.1 Select and use the appropriate tool to determine measurements of length, area, perimeter, volume, and angle size.
4.4.2 Use estimation strategies in working with quantities, measurement, computation, and problem solving.
4.4.3 Apply a variety of techniques, tools, and formulas to determine measurements.
4.4.4 Know and use units of time, money, and temperature.

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Standard 5: ALGEBRA, FUNCTIONS, AND PATTERNS

Students use algebraic concepts, functions, patterns, and relationships to solve problems.

4.5.1 Understand when a simple pattern exists, identify the rule that generates the pattern, and use that information to solve problems.
4.5.2 Represent and describe mathematical relationships using symbols for variables.
4.5.3 Solve problems with unknown variables.
4.5.4 Use basic counting strategies to determine all possible outcomes.

Standard 1: NUMBER AND OPERATION

Students understand and use basic and advanced concepts of number and number systems.

Benchmarks

4.1.1 Construct and interpret number meanings through real-world experiences.
4.1.2 Understand the characteristics and properties of our numeration system.
4.1.3 Understand how arithmetic operations are related to one another in addition, subtraction, multiplication, and division.
4.1.4 Rename, order, and compare numbers.
4.1.5 Know and use basic facts and computational algorithms for whole numbers, fractions and decimals.
4.1.6 Use estimation strategies in working with quantities, measurement, computation, and problem solving.
4.1.7 Understand and communicate strategies to solve a wide variety of problems.

Examples of Specific Knowledge that Support the Standard and Benchmarks

4.1.1 grouping, ordering, one-to-one correspondence, fractions, decimals, percents
4.1.2 place value, grouping, ordering base 10, fractions, decimals, percents, standard numbers, expanded numbers, ordinal numbers, cardinal numbers, odd and even numbers
4.1.3 ordering, addition (addend, sum), subtraction (minuend, subtrahend, difference) multiplication (multiplier, product), division (divisor, dividend, quotient)
4.1.4 relationships, comparing, relative size, most/least, greater than/less than/equal to, sorting
4.1.5 computation, addition, subtraction, multiplication, division of whole numbers and decimals, addition and subtraction of like fractions
4.1.6 know when to use paper-pencil, mental math, calculator or manipulatives
4.1.7 guess/check, work backwards, draw diagram, use objects, pertinent information, irrelevant information

Examples of Activities that Support the Standard and Benchmarks

4.1.1 Students count the days in school using a variety of manipulatives, grouping by 2's, 5's, 10's etc.
4.1.1 Students trade coin values to demonstrate the connections between the base 10 number system and the base 10 monetary system.
4.1.2 Students represent numerals using place value materials such as Base 10 blocks, determining greater than, less than, or equal to (<, >, =).
4.1.3 Students use two different colored manipulatives to demonstrate a fact family.

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Examples of Activities that Support the Standard and Benchmarks

4.1.4 Students identify the amount of money in the teacher's pocket by asking questions that can only be answered by yes or no (e.g., "Is it odd?" "Is it greater than 50 cents?")

4.1.5 A student draws five cards from a deck of 40 cards (0 to 9 deck). Each student solves the problem by placing operational symbols to make the first four cards equal the value of the last card.

4.1.6 Using a 100 chart, students find the sum or difference of two numbers. Students are given a number as the starting point and another number to add to it. Students find the sum by separating the given number into tens and ones and moving along the rows and columns of the chart for tens and ones respectively. For example, the starting number is 25 and 12 is the number to be added. The sum is obtained by moving to the right two places and down one place to land on 37, the sum of 25 and 12. Subtraction is done by moving in the opposite directions (left for ones and up for tens).

4.1.7 Students demonstrate with manipulatives their understanding of the four basic operations. They transfer their work to graph paper and generate a number sentence that corresponds to that algorithm. (This activity also applies to Benchmark 4.1.6)

4.1.7 Students work in groups of three with one using a calculator, one using pencil and paper, and one using mental math. Individuals within the groups race to solve problems presented by the teacher. As a class, students discuss which method is most efficient for each problem.

4.1.7 Students generate word problems and compile them into a class book to take home and solve with their families. As a whole class, they share their strategies for solving the problems and identify the many ways to solve each problem.

4.1.7 Students create a multi-step problem that includes irrelevant information and then exchange problems with other peers to solve.

Standard 2: GEOMETRY AND SPATIAL SENSE

Students understand and apply geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations.

Benchmarks

4.2.1 Know the characteristics of two- and three-dimensional shapes.

4.2.2 Understand how two- and three-dimensional shapes can be changed by combining and dividing.

4.2.3 Understand that geometry is found within and outside mathematics.

Examples of Specific Knowledge that Support the Standard and Benchmarks

4.2.1 shapes (e.g., triangle, square, rectangle, circle, hexagon), two-dimensional, three-dimensional, polygons (number of sides, number of angles), geometric solids (i.e., number of faces, number of edges, number of vertices), attributes (i.e., size, color, shape), quadrilateral, parallelogram, equilateral, obtuse, acute, right, line segment, intersecting, perpendicular, parallel, vertex, symmetrical, congruent, ray

4.2.2 transformations, flips, turns, slides, rotation, reflection, symmetrical, congruent

4.2.3 visualizing, spatial reasoning (e.g., above, below, behind, between, inside)

Examples of Activities that Support the Standard and Benchmarks

4.2.1 Each small group of students is given a piece of yarn 15 feet long. The group is asked to form a variety of shapes, keeping both hands on the yarn at all times. Students determine if they can make the shape and then draw and describe the characteristics of that shape, using pencil, paper, or computer, including other names for the shape.

4.2.1 Students build shapes on geoboards and predict how many shapes they can make by subdividing or combining their original shapes. They build the structures and check out

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their answer.

Examples of Activities that Support the Standard and Benchmarks

4.2.2 Students are given a packet of pattern blocks (hexagon, rhombus, triangle and trapezoid shapes only). Students work in pairs with one student selecting a shape and the other student selecting two or more shapes that equal the first shape.

4.2.3 Students take a walk outside the classroom and record the geometric shapes they find.

Standard 3: DATA ANALYSIS, STATISTICS AND PROBABILITY

Students use data collection and analysis techniques, statistical methods, and probability to solve problems.

Benchmarks

4.3.1 Collect, organize, and display data.
4.3.2 Formulate and solve problems that involve data.
4.3.3 Draw conclusions based on probability.
4.3.4 Use technology and materials as tools to display data.

Examples of Specific Knowledge that Support the Standard and Benchmarks

4.3.1 grid (vertical axis, horizontal axis, coordinates), histogram, line graph, bar graph, circle graph, floor graph, pictograph, glyph, Venn diagram, spread sheets, sample, sets, intersection, union
4.3.2 mean, mode, median, range, interpret, draw conclusions, statistics
4.3.3 classify, sort, probability, chance
4.3.4 spread sheet, computer, floor graph, Unifix stacks, sticky note graphs

Examples of Activities that Support the Standard and Benchmarks

4.3.1 Students individually or as a whole group sprout three different types of seeds in various ways (e.g., a damp paper towel in a baggie, moistened sponge, soil). They predict which seed will sprout first in each situation. Results can be recorded and interpreted through the use of a daily journal, illustrations, and individual or class graphs.

4.3.1 Students sort objects or pictures of objects that have one or more attributes, such as color, shape, or size, using a Venn diagram.

4.3.2 Students collect Hot Lunch count for individual classes for a week. Using this information, they determine favorite or least favorite foods. Other interpretations could include food preferences by different ages of students, specific day of week comparisons and daily absences. The students could determine the weekly average of individual classes. (This activity also addresses Benchmark 4.3.1.)

4.3.2 Students predict which color M & M they are most likely to choose from a bag. They collect and represent data using a variety of techniques. Examples of useful information might include average number of color or how many colors are represented. Groups make a presentation to the class, using computer, graphs, etc. (This activity also addresses Benchmark 4.3.1 and 4.3.4.)

4.3.3 Each student tosses a coin ten times and records the results on graph paper. They compare individual results and compile information on a large class graph.

4.3.3 Students bring a favorite toy, create a class set, then sort and categorize items. The class graphs the data and students describe and analyze the information collected. (This activity also addresses Benchmark 4.3.1.)

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4.3.4	Students use a variety of ways to graph the results of various surveys that they choose to conduct. (Some ways and materials they might use to graph the results include hex-a-link cubes of various colors, straws in baggies, tally graphs, pattern block graphs, Popsicle sticks in plastic cups, beans glued on a bar graph, computer software.) [This activity also addresses Benchmark 4.3.1.]

Standard 4: MEASUREMENT

Students use concepts and tools of measurement to describe and quantify the world.

Benchmarks	
4.4.1	Select and use the appropriate tool to determine measurements of length, area, perimeter, volume, and angle size.
4.4.2	Use estimation strategies in working with quantities, measurement, computation and problem solving.
4.4.3	Apply a variety of techniques, tools, and formulas to determine measurements.
4.4.4	Know and use units of time, money, and temperature

Examples of Specific Knowledge that Support the Standard and Benchmarks	
4.4.1	circumference, radius, diameter, capacity, compass, protractor, rulers, dot paper, geoboard, grid paper, Unifix cubes, links
4.4.2	about, almost, less than, more than, equal to
4.4.3	scales, balance scales, rulers / links, graduated cylinder / scoops, clocks, yard / meter / baseball bat, inch / centimeter / link, quart / liter / pitcher, pound / kilogram / brick, standard and non-standard tools, formulas for finding area and perimeter, conversions (inches to feet, minutes to hours, etc.) proportion (map reading)
4.4.4	coins (recognize, count coins, count change), tell time (analog, digital), days, months, year, Celsius, Fahrenheit

Examples of Activities that Support the Standard and Benchmarks	
4.4.1	Students construct a polygon on a geoboard or dot paper. They identify the angles, determine the area and perimeter, and explain in writing why their answer is correct.
4.4.2	Students guess the metric and standard weight of a jar of jellybeans. They can also guess and check how many beans are in the jar using a variety of different items in the jar according to weight, size, etc.
4.4.3	Students work in groups to determine the appropriate tool to measure a variety of objects, finding attributes such as length, weight, area, volume, etc.
4.4.4	Using catalogues and newspapers, students simulate a shopping trip. They have $25.00 to spend and three and a half hours (9 a.m. to 12:30 p.m.) to spend it. Students document how they spent their money and how much time it took them to complete their shopping trip. (They must account for the time it takes to travel between stores and to do the shopping.)

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Standard 5: ALGEBRA, FUNCTIONS, AND PATTERNS

Students use algebraic concepts, functions, patterns, and relationships to solve problems.

Benchmarks

4.5.1 Understand when a simple pattern exists, identify the rule that generates the pattern, and use that information to solve problems.
4.5.2 Represent and describe mathematical relationships using symbols for variables.
4.5.3 Solve problems with unknown variables.
4.5.4 Use basic counting strategies to determine all possible outcomes.

Examples of Specific Knowledge that Support the Standard and Benchmarks

4.5.1 identify, describe, extend and create patterns, compare attributes, people patterns
4.5.2 ¹ (does not equal), < (is less than), > (is greater than), = (is equal to), coordinate grid, use of brackets, equality, inequality
4.5.3 a+7 = 10 È a = 3, models, known facts, properties, relationships, equations, addends, factors
4.5.4 tree diagrams, permutations, combinations, sequence, if-then problems

Examples of Activities that Support the Standard and Benchmarks

4.5.1 Students interact and predict a pattern which grows daily and changes monthly. For example, September could be an AABAAB pattern, October could be ABABAB, November could be ABBABB, etc. These patterns should be developed and displayed throughout the room.
4.5.1 Students form possible people patterns: the first child stand, the second sit, the third stand, the fourth sit, etc. resulting in SsSsSsSs. They create some people patterns on their own.
4.5.2 Students are grouped into two sections, Group A and Group B with Group B containing more students than Group A. A hula hoop is placed between the two groups. The students must decide how many students to place in the hula hoop so that the number of students in Group A plus the number of students in the hula hoop will be the same as the number of students in Group B. The students then write a number sentence to represent the situation. Further discussion might include how to change this sentence into a subtraction sentence.
4.5.3 Students use pattern blocks to solve the following problem: Each triangle equals two squares. Two squares equal a circle. How many triangles equal a circle?
4.5.4 Students use three different colored manipulatives to determine how many different ways they can arrange the objects in a row. They explain in writing how they know that they have found all the different ways.

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Summary of Grades 5 - 8 Benchmarks

Standard 1: NUMBER AND OPERATION

Students understand and use basic and advanced concepts of number and number systems.

8.1.1 Understand numbers, number systems and ways of representing numbers.
8.1.2 Apply number theory concepts in mathematical problems.
8.1.3 Determine appropriate information and methods to solve problems.
8.1.4 Compute with real numbers using appropriate computational methods for given situations.
8.1.5 Apply appropriate estimation strategies to determine if a solution is reasonable.

Standard 2: GEOMETRY AND SPATIAL SENSE

Students understand and apply geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations.

8.2.1 Understand the relationships between two and three-dimensional models using a variety of materials and tools.
8.2.2 Know, and reason informally about, properties of two and three-dimensional figures.
8.2.3 Know the components of the coordinate plane.
8.2.4 Determine perimeter, circumference, and area in two dimensions, and surface area and volume in three dimensions.
8.2.5 Use transformations and symmetry to investigate similar figures.

Standard 3: DATA ANALYSIS, STATISTICS, AND PROBABILITY

Students use data collection and analysis techniques, statistical methods, and probability to solve problems.

8.3.1 Collect, read, and display data using appropriate techniques and technology.
8.3.2 Display and use measures of central tendency and measures of variability.
8.3.3 Evaluate arguments that are based on statistical claims.
8.3.4 Identify basic trends in tables and graphs and use these trends to make predictions.
8.3.5 Determine probabilities through experiments or simulations.
8.3.6 Understand and apply the basic notions of probability.
8.3.7 Use counting strategies to determine all the possible outcomes.

Standard 4: MEASUREMENT

Students use concepts and tools of measurement to describe and quantify the world.

8.4.1 Select appropriate units and scale to estimate and measure.
8.4.2 Select and use appropriate measurement unit and tools to solve problems.
8.4.3 Use formulas and procedures to solve problems involving measurement.

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Standard 5: ALGEBRA, FUNCTIONS, AND PATTERNS

Students use algebraic concepts, functions, patterns, and relationships to solve problems.

8.5.1 Find, represent, describe, and analyze patterns, functions, and relations using tables, graphs, verbal descriptions, and standard algebraic notation.
8.5.2 Understand the concepts of functions, variables, expressions, equations, and inequalities.
8.5.3 Solve linear equations, inequalities, and systems of equations in problem-solving situations using a variety of methods and a variety of tools.

Standard 1: NUMBER AND OPERATION

Students understand and use basic and advanced concepts of number and number] systems.

Benchmarks

8.1.1 Understand numbers, number systems, and ways of representing numbers.
8.1.2 Apply number theory concepts in mathematical problems.
8.1.3 Determine appropriate information and methods to solve problems.
8.1.4 Compute with real numbers using appropriate computational methods for given situations.
8.1.5 Apply appropriate estimation strategies to determine if a solution is reasonable.

Examples of Specific Knowledge that Support the Standard and Benchmarks

8.1.1 fractions; decimals; integers; scientific notation; ratio; radical; place value; associative, commutative, and distributive properties; properties of one and zero; improper and proper fractions
8.1.2 lowest common multiples, greatest common factors, prime, composite, factor tree, divisibility rules, multiples, factors, prime factorization
8.1.3 extraneous information, guess and check, use of diagrams, tables and charts
8.1.4 add, subtract, multiply, divide real numbers; order of operations
8.1.5 rounding (to the nearest), front end estimation, compatible numbers, clusters, rounding up and down, truncate, trial and error, reasonable solution

Examples of Activities that Support the Standard and Benchmarks

8.1.1 Students represent different fractions, decimals, and percents using pattern blocks. Working in pairs, one student builds the model of a fraction, decimal or percent with the pattern blocks. The partner describes verbally or in written form a related fraction, decimal or percent.
8.1.1 Students, using calculators, explore addition and multiplication of odd and even numbers. The students, in small groups, record, discuss and compare results. The group should form conjectures from their findings.
8.1.1 Students respond in written form to questions such as: How can you get a smaller answer when you multiply? How can you get a larger answer when you divide? Students should be allowed to explore with calculators before answering these questions.
8.1.1 Students make a scale drawing of their favorite room in their home.
8.1.2 Students, working with partners or in small groups, develop a game called “What is My Number?” They write clues for their secret number using vocabulary, such as prime, factor, multiples and squared. They can trade their game with other groups so each group has a new game to play.

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Examples of Activities that Support the Standard and Benchmarks

8.1.3 Students discuss what information is needed and not needed when given a variety of story problems. After they have decided on the needed information, they explore the different strategies: find a pattern, guess and check, make a table, draw a diagram, and use a simpler problem.

8.1.4 Students, working in teams, use bingo game boards with products of whole numbers, fractions, decimals, etc. They choose from a list of factors (given to them by the teacher) to produce the products on the bingo boards. Students use estimating skills when choosing the numbers and then multiply to find exact answers. The team that captures four cells in a row is the winning team.

8.1.4 Students solve the following problem: Your bill at a restaurant is $26. Use the distributive property to determine a 15% tip. [Tip = (.1 x meal cost) + 1/2 (.1 x meal cost)]

8.1.5 Students estimate amounts such as how much they spend on candy each week, how many hours they spend watching TV, etc.

Standard 2: GEOMETRY AND SPATIAL SENSE

Students understand and apply geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations.

Benchmarks

8.2.1 Understand the relationships between two- and three-dimensional models using a variety of materials and tools.

8.2.2 Know, and reason informally about, the properties of two- and three-dimensional figures.

8.2.3 Know the components of the coordinate plane.

8.2.4 Determine perimeter, circumference, and area in two dimensions, and surface area and volume in three dimensions.

8.2.5 Use transformations and symmetry to investigate similar figures.

Examples of Specific Knowledge that Support the Standard and Benchmarks

8.2.1 polygon, solid, cones, prisms, pyramid, bisect, protractor, compass, face, cylinder

8.2.2 angles, hypotenuse, congruent, similar, ray, plane, edge, vertex, parallel, perpendicular, legs, face diagonal, obtuse, isosceles, scalene, right, acute, straight, equilateral, point, lines, segment, skew, Pythagorean Theorem

8.2.3 quadrants, origin, coordinates, x- and y- axes

8.2.4 radius, diameter, chord, square and cubic units, arc, faces

8.2.5 transformations, rotation (turn), reflections (flip), translations (slide), tessellations

** Indicates the word or phrase is defined in the Glossary*

Examples of Activities that Support the Standard and Benchmarks

8.2.1 Each student is provided a bag that has multi-link cubes inside. One student builds a figure in the bag. Without looking, the second student feels the figure built by the first student and then tries to build an identical figure.

8.2.2 Students create two and three-dimensional figures using toothpicks and mini marshmallows. Students discuss, in small groups, questions such as: How many toothpicks did you use? How many marshmallows did you use? Are there any parallel lines, perpendicular lines, or skew lines? How many edges are there? How many vertices are there? Students then draw their figures on isometric dot paper.

8.2.3 Students play "Sink the Ship" using coordinate graph paper. They "hide" their battleships which are represented by dots on the grid lines. Students keep their papers covered so their opponent cannot see where they have marked their papers. Students take turns naming ordered pairs to locate the opponent's ship. The object of the game is to sink the opponent's battleship.

8.2.4 Students find the surface area and volume of three-dimensional figures. Figures might include cones, cylinders, cubes, and rectangular prisms. Students are given the cost of cardboard per square inch and use that information to determine which figures would be most cost effective in packaging.

8.2.5 Students, working in pairs, use geoboards to construct a polygon. The partners make a reflection, translation, or rotation of the given polygon. Related discussion should include whether the second polygon meets the definition of the transformation.

Standard 3: DATA ANALYSIS, STATISTICS, AND PROBABILITY

Students use data collection and analysis techniques, statistical methods, and probability to solve problems.

Benchmarks

8.3.1 Collect, read and display data using appropriate techniques and technology.
8.3.2 Display and use measures of central tendency and measures of variability.
8.3.3 Evaluate arguments that are based on statistical claims.
8.3.4 Identify basic trends in tables and graphs and use these trends to make predictions.
8.3.5 Determine probabilities through experiments or simulations.
8.3.6 Understand and apply the basic notions of probability.
8.3.7 Use counting strategies to determine all the possible outcomes.

Examples of Specific Knowledge that Support the Standard and Benchmarks

8.3.1 bar graph, line graphs, scatter plots, box-and-whisker plots, stem-and-leaf plots, circle graphs, data collection, sample, outlier
8.3.2 mean, median, mode, range, frequency
8.3.3 graphs, charts, tables, misleading graphs/tables/charts, extrapolate, interpolate
8.3.4 hypothesis, conclusion
8.3.5 chance, dependent events, independent events, outcome
8.3.6 sample, population, random
8.3.7 probability, permutations, combinations, factorials, Venn Diagram, tree diagram

** Indicates the word or phrase is defined in the Glossary*

Examples of Activities that Support the Standard and Benchmarks

8.3.1 Students survey friends using the question: "How many hours a week do you watch TV?" Groups of students compile their findings and display this data in two different types of graphs (e.g., line graphs, circle graphs, stem and leaf plot).

8.3.2 Students work with partners to measure one another's height in centimeters. These heights are recorded on the board. Each student is to find the mean, mode, range, and median of the heights.

8.3.3 Students display the same information in different graphical forms. Students decide which graph displays the information the best or the most accurately.

8.3.4 Students collect samples of graphs of statistical data from newspapers and write a two paragraph article related to one of the graphs.

8.3.5 Students, working in groups, have a paper bag containing an assortment of ten colored tiles. Every student in each group, one at a time, draws out a tile, records the color, and replaces the tile. This is repeated ten times. Students figure the probability of drawing each color (number of times color drawn/number of draws).

8.3.6 Students flip two coins 50 times and record the results. They determine how the probability of getting two heads compares with the theoretical probability of getting two heads.

8.3.7 Students solve problems such as: Mary tells a secret to Jon and Tasha on Monday. Jon and Tasha each tell two friends the secret on Tuesday. The friends of Jon and Tasha then each tell two of their friends on Wednesday. If this pattern continues, how many new friends will hear the secret on Saturday? How many people know the secret on Saturday? In how many days will everyone at school know the secret if there are 780 students in the school?

8.3.7 Students write possible arrangements of sets with a different number of objects in them, such as the letters in their name, six people standing in a row, or the first four finishers in a race. They draw conclusions about finding the number of possible outcomes for a number of objects.

Standard 4: MEASUREMENT

Students use concepts and tools of measurement to describe and quantify the world.

Benchmarks

8.4.1 Select appropriate units and scale to estimate and measure.

8.4.2 Select and use appropriate measurement unit and tools when solving problems.

8.4.3 Use formulas and procedures to solve problems involving measurement.

Examples of Specific Knowledge that Support the Standard and Benchmarks

8.4.1 angles, perimeter, area, surface area, volume, indirect measurements

8.4.2 degrees, metrics, pounds, tons, linear units, square units, cubic unites, ruler, meter stick, yard stick, compass, protractor, standard units, non-standard units (e.g., arm or step length), concepts of precision and significant digits

8.4.3 perimeter, area, volume, surface area, length, width, conversion of units of measure

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Examples of Activities that Support the Standard and Benchmarks

8.4.1 Students, working in pairs, use a map of North Dakota to find its area. First they do a rough estimate and then refine the estimate using the formula for area of a rectangle. Students explain their strategies.

8.4.2 Students cut a sheet of graph paper in half horizontally and roll each half to make two cylinders. One will be tall and skinny, and the other short and squatty. Students answer the following questions: Are the volumes of the cylinders the same? If not, which will hold more? The volumes could be measured with a material, such as rice. The volumes also could be calculated.

8.4.2 Students decide how many Big Macs would fit in their classroom.

8.4.3 Students find the area of irregular shaped figures, such as leaves. Give students a packet containing a variety of objects, including string, graph paper, dot paper, scissors, marker, and transparency paper to measure area.

8.4.3 Students find the surface area of the classroom. Using this information, the students find the cost to paint or wallpaper the classroom.

Standard 5: ALGEBRA, FUNCTIONS, AND PATTERNS

Students use algebraic concepts, functions, patterns, and relationships to solve problems.

Benchmarks

8.5.1 Find, represent, describe, and analyze patterns, functions, and relations using tables, graphs, verbal descriptions, and standard algebraic notation.

8.5.2 Understand the concepts of variables, expressions, equations and inequalities.

8.5.3 Solve linear equations, inequalities, and systems of equations in problem-solving situations using a variety of methods and a variety of tools.

Examples of Specific Knowledge that Support the Standard and Benchmarks

8.5.1 number sequences, inductive reasoning, domain, range, variation, independent and dependent variable

8.5.2 inequalities, simplify, evaluate, polynomial, solution, like terms, equivalent equation, equivalent systems

8.5.3 substitution, intersection of two lines, intercepts, solution set, slope, rate of change, graphing, coordinate grid, Fibonacci sequences, Pascal's triangle, find algebraic representation of rule of sequences, rate-time-distance problems, interest problems, arithmetic and geometric progressions

Examples of Activities that Support the Standard and Benchmarks

8.5.1 Students are given the sequence 1,1,2,3,5,8,... and are directed to find the next three terms. Students write a formula to identify any term and discover a way to find the sum of the terms.

8.5.2 Students find all the diagonals for a regular polygon (e.g., an equilateral triangle has none, a square has two, a regular pentagon has ?...). Students determine if there is a formula that could be used for a 100-gon.

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Examples of Activities that Support the Standard and Benchmarks

8.5.3 Students examine the beginning of the Fibonacci sequence (1,1,2,3,5,8,13,…) or the first four rows of Pascal's triangle (row 1: 1, row 2: 1 1, row 3: 1 2 1, row 4: 1 3 3 1). Students find the next five numbers in the Fibonacci sequence or the next three rows of Pascal's Triangle. They discuss how the numbers in either pattern are related. Students can make a sequence of their own for fellow students to solve.

8.5.4 Students solve a problem such as: John, Mary, and Bill work on an assembly line. They are paid a weekly base salary and then an additional amount for each widget they make. Use the equation Total Salary = (Weekly base) + (Number of pieces) x (Pay per piece) to solve the following problem: Three weeks ago John received $125 in total pay of which $100 represents his base pay. If he is paid $.25 per widget, how many widgets did John make? Mary was paid $200. She made exactly 50 widgets. If she is paid $.25 per widget, how much is her weekly base pay? If Bill receives a base pay of $50 and $.15 per widget, graph his possible weekly incomes based on widgets produced.

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Summary of Grades 9 - 12 Benchmarks

Standard 1: NUMBER AND OPERATION

Students understand and use basic and advanced concepts of number and number systems.

12.1.1 Know and use the real number system, its subsets and properties.
12.1.2 Identify complex numbers and understand their relevance in solving equations.
12.1.3 Use basic set relations and operations with appropriate notation.
12.1.4 Understand the meaning of operations and how they relate to each other.
12.1.5 Apply advanced estimation skills and appropriate computational methods to attain reasonable solutions.
12.1.6 Understand the properties and basic theorems of roots, exponents, and logarithms.

Standard 2: GEOMETRY AND SPATIAL SENSE

Students understand and apply geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations.

12.2.1 Understand and apply the properties of two- and three-dimensional figures.
12.2.2 Construct basic geometric figures using appropriate tools.
12.2.3 Understand the concepts of congruence, similarity, and symmetry.
12.2.4 Apply transformations to basic shapes.
12.2.5 Apply the Pythagorean Theorem to solve problems.
12.2.6 Apply basic trigonometric ratios to solve real-world problems.
12.2.7 Apply measurements and formulas in computations of perimeter, area, and volume.
12.2.8 Generate geometric conjectures inductively and validate them deductively.
12.2.9 Apply geometric properties to a coordinate system.

Standard 3: DATA ANALYSIS, STATISTICS, AND PROBABILITY

Students use data collection and analysis techniques, statistical methods, and probability to solve problems.

12.3.1 Design, implement, and present statistical studies.
12.3.2 Sample data and understand the role of sampling in data analysis.
12.3.3 Use counting strategies.
12.3.4 Calculate theoretical and experimental probabilities.
12.3.5 Calculate and interpret measures of central tendency and variance.
12.3.6 Use regression techniques to determine and interpret the curve of best fit.
12.3.7 Draw inferences and predict outcomes from data expressed in a variety of ways.

Standard 4: MEASUREMENT

Students use concepts and tools of measurement to describe and quantify the world.

12.4.1 Understand attributes, units, and systems of measurement.
12.4.2 Apply a variety of techniques, tools, and formulas to determine measurements.
12.4.3 Measure physical quantities and determine measurement error.
12.4.4 Use estimation in the measurement process.

** Indicates the word or phrase is defined in the Glossary*

Standard 5: ALGEBRA, FUNCTIONS, AND PATTERNS

Students use algebraic concepts, functions, patterns, and relationships to solve problems.

12.5.1 Use algebraic procedures to manipulate mathematical expressions.
12.5.2 Solve equations, inequalities, and systems.
12.5.3 Represent and describe relations algebraically, numerically, and graphically.
12.5.4 Create, manipulate, and apply matrices to real-life situations.
12.5.5 Develop and analyze a variety of algorithms.
12.5.6 Understand and apply the process of recursion.
12.5.7 Use patterns and functions to model problems.
12.5.8* Understand the basic ideas about convergence, limit of functions, and infinite series.

Standard 1: NUMBER AND OPERATION

Students understand and use basic and advanced concepts of number and number systems.

Benchmarks

12.1.1 Know and use the real number system, its subsets and properties.
12.1.2 Identify complex numbers and understand their relevance in solving equations.
12.1.3 Use basic set relations and operations with appropriate notation.
12.1.4 Understand the meaning of operations and how they relate to each other.
12.1.5 Apply advanced estimation skills and appropriate computational methods to attain reasonable solutions.
12.1.6 Understand the properties and basic theorems of roots, exponents, and logarithms.

Examples of Specific Knowledge that Support the Standard and Benchmarks

12.1.1 natural numbers, whole numbers, integers, rational numbers, irrational numbers, transcendental numbers; properties of closure, commutative, associative, distributive, identity, and inverse; properties of reflexive, symmetric, and transitive
12.1.2 complex, real, imaginary, rational, irrational, integers, whole, natural, *conjugate
12.1.3 sets, Venn diagram, intersection, union, subsets, elements
12.1.4 exponential, *logarithmic, complex numbers, vector addition, scalar, scalar multiplication, matrix (addition, subtraction, multiplication)
12.1.5 Fermi problems, absolute and relative error
12.1.6 Divisibility, remainders, factors, multiples, prime, relatively prime

Examples of Activities that Support the Standard and Benchmarks

12.1.1 Students take an algebra problem that has been solved and identify the property that justifies each step.
12.1.2 Students draw and label a Venn diagram to represent the complex number system. They write a specific example in each area of the diagram.
12.1.2 Students solve problems such as: Solve $2x^2 + 3x + 4 = 0$ over the set of complex numbers. They graph this equation and explain in writing its relationship to their complex solution.
12.1.3 Students take given sets such as A = {-5, -2, 0, 1, 6}, B = {-3, -2, -1, 0, 1}, and C = {whole numbers} to determine A È B and A Ç C and answer the question "Is (A Ç C) Ì C?"
12.1.4 Given two matrices $A = \begin{bmatrix} 1 & 3 & 6 \\ -2 & 5 & 2 \\ 0 & -4 & 7 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -8 & 4 \\ 1 & 3 & -5 \\ 6 & -4 & 0 \end{bmatrix}$ and Find A + B, AB, and BA.

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Examples of Activities that Support the Standard and Benchmarks

12.1.5 Students work in groups to determine an approximate amount spent by the sophomore class on a particular day. Students explain how they obtained their approximation. 12.1.5 Students solve such problems as: The national debt in 1993 was about six trillion (6,000,000,000,000) dollars. How long do you think it would take to spend this amount if you were to spend a million dollars an hour until the money was gone? Find how reasonable your estimate was. 12.1.6 Students determine the largest *n* value such that $2n = 20!$

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Standard 2: GEOMETRY AND SPATIAL SENSE

Students understand and apply geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations.

Benchmarks

12.2.1 Understand and apply the properties of two- and three-dimensional figures.
12.2.2 Construct basic geometric figures using appropriate tools.
12.2.3 Understand the concepts of congruence, similarity, and symmetry.
12.2.4 Apply transformations to basic shapes.
12.2.5 Apply the Pythagorean Theorem to solve problems.
12.2.6 Apply basic trigonometric ratios to solve real-world problems.
12.2.7 Apply measurements and formulas in computations of perimeter, area, and volume.
12.2.8 Generate geometric conjectures inductively and validate them deductively.
12.2.9 Apply geometric properties to a coordinate system.

Examples of Specific Knowledge that Support the Standard and Benchmarks

12.2.1 polyhedron, polygon, perspective drawing
12.2.2 compass, protractor, computer software, Mira boards
12.2.3 rotational, line, and plane symmetries
12.2.4 translation, reflection, rotation, dilation, isometry
12.2.5 legs, hypotenuse
12.2.6 sine, cosine, tangent
12.2.7 length, width, height, altitude, apothem
12.2.8 hypothesis, conclusion, conditional, converse, inverse, contrapositve, *truth tables
12.2.9 rectangular coordinate system, *polar coordinate system

Examples of Activities that Support the Standard and Benchmarks

12.2.1 Students create as many non-congruent, two-dimensional figures (nets) as possible that can be folded to form a cube.
12.2.1 Students make a perspective drawing of a variety of three-dimensional objects.
12.2.2 Students find the location to build a radio tower that is equidistant from Jamestown, Grand Forks and Fargo, using ruler and compass or dynamic geometry software.
12.2.2 Students construct a right triangle and then construct a square on each side of the triangle with side length equal to each edge of the triangle. They calculate the area of each or cut and paste to illustrate the Pythagorean Theorem ($a2 + b2 = c2$).
12.2.3 Given two figures, students identify as many relationships between them in terms of congruence, similarity, and symmetry.
12.2.4 Students create a design to tile a floor or a wall pattern using tessellations and appropriate technology.
12.2.5 Students write their own right triangle application problems. They should provide a complete solution for each problem, including a diagram. Students share their problems with classmates.

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Examples of Activities that Support the Standard and Benchmarks

12.2.6 Students determine unknown dimensions by indirect measurement. They compare the results with those obtained by direct measurement, commenting on the methods and the associated error.

12.2.7 Students design the most cost effective container for shipping 50 loose golf balls.

12.2.7 Students bring empty containers (e.g., Quick containers, coffee cans, spice cans). Measure the dimensions of the container and find its capacity. Students can then fill the container with water to verify their calculated volume.

12.2.8 Students construct six different isosceles triangles. Then they construct an altitude from the vertex formed by the two congruent sides using appropriate technologies. They measure the angles and sides of each figure. Based on these measurements, they make as many conjectures as possible about properties of isosceles triangles. They provide deductive arguments for as many of the conjectures as they can.

12.2.9 Given three specific non-collinear points, students find the area and perimeter of the figure formed by these points.

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Standard 3: DATA ANALYSIS, STATISTICS, AND PROBABILITY

Students use data collection and analysis techniques, statistical methods, and probability to solve problems.

Benchmarks

12.3.1 Design, implement, and present statistical studies.
12.3.2 Sample data and understand the role of sampling in data analysis.
12.3.3 Use counting strategies.
12.3.4 Calculate theoretical and experimental probabilities.
12.3.5 Calculate and interpret measures of central tendency and variance.
12.3.6 Use regression techniques to determine and interpret the curve of best fit.
12.3.7 Draw inferences and predict outcomes from data expressed in a variety of ways.

Examples of Specific Knowledge that Support the Standard and Benchmarks

12.3.1 sample size, population, descriptive statistics, *inferential statistics
12.3.2 random sample
12.3.3 factorial, permutations, combinations
12.3.4 sample space, event, probability of an event
12.3.5 mean, median, mode, range, quartiles, standard deviation
12.3.6 correlation, *bivariate plot, *least squares
12.3.7 statistic, normal probability distribution, *binomial probability distribution, *parameter, *chi-square and t-tests, *random variable, *hypothesis testing, *sampling techniques, *t-score, *z-score, *standard score

Examples of Activities that Support the Standard and Benchmarks

12.3.1 Students generate interesting problems that can be solved by collecting data. Students collect, organize and describe the data. Then they make interpretations and report their findings.

12.3.1 Students gather data on the time of sunrise and sunset over several months. They use these data to find a relationship and make predictions.

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Examples of Activities that Support the Standard and Benchmarks

12.3.2 Students simulate a “catch and release” program used to estimate the number of fish in a body of water by putting in a quantity of white beans and 20 red beans in a container. The total number of white and red beans represents the total number of fish in the water. The number of red beans represents the number of tagged fish. Each group takes a 1/4 cup measure of beans and counts the number of beans in the sample as well as how many are red. Replace all beans into the container, mix thoroughly and repeat the procedure. The students are divided into three groups (Sections A, B, and C). Section A takes three samples, Section B takes six samples, and Section C takes ten samples. Each group finds the average number of total beans drawn to estimate the number of beans in the container. Students discuss the effects, if any, of the difference in the number of samples taken.

12.3.3 Five students from the class are chosen and asked to stand together where everyone in the room can see them. The class responds to the question, "In how many ways can these students stand in a straight line?" "How many ways can three of the five students be arranged in a line?" "How many ways can a committee of three students be chosen from among these five students?"

12.3.3 Given the following situation, students find how many different pizzas can be made using at most four toppings: A pizza shop lists 11 different toppings (other than just plain cheese) that it can put on pizzas.

12.3.4 Students’ roll a pair of die 100 times and record the sum of the uppermost numbers each time. They use the results to find the empirical probability of rolling a seven with these dice and determine how the experimental probability compares with the theoretical probability of rolling a seven.

12.3.5 A jar is filled with M&Ms. Each student in the class guesses how many are in the jar. Then the students describe the distribution of guesses graphically and by giving summary statistics, including mean, median, mode, range, variance, and standard deviation.

12.3.6 Students find examples of circles from everyday life and measure the radii and circumference of each. They draw the circles on centimeter grid paper and estimate the area. They produce a scatter plot of diameter versus circumference and radius versus area and find the equations of the curves of best fit using appropriate technology.

12.3.7 A jar is filled with M&M’s . Each student in the class guesses how many M&Ms are in the jar. After determining the mean from the guesses, students calculate the percentages of guesses that are within one and two standard deviations of the mean. Students comment on the degree to which these guesses appear to be normally distributed.

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Standard 4: MEASUREMENT

Students use concepts and tools of measurement to describe and quantify the world.

Benchmarks

12.4.1 Understand attributes, units, and systems of measurement.

12.4.2 Apply a variety of techniques, tools, and formulas to determine measurements.

12.4.3 Measure physical quantities and determine measurement error.

12.4.4 Use estimation in the measurement process.

Examples of Specific Knowledge that Support the Standard and Benchmarks

12.4.1 unit of measure, rate of change, approximation
12.4.2 radian and degree measures, slope, velocity
12.4.3 metric and English systems, degrees, minutes, seconds, longitude, latitude
12.4.4 units of measure

Examples of Activities that Support the Standard and Benchmarks

12.4.1 Students graph the distances the planets are from the sun using a linear scale and a logarithmic scale.
12.4.2 Students estimate the height of a multi-story building by a variety of techniques (e.g., trigonometric methods, shadow and proportions, comparative heights).
12.4.3 Using only tape measures, groups of students mark out various quadrilaterals on the sport field. The groups report to the class on their methods and possible errors.
12.4.4 Students estimate how many phone books the phone company should print for their town. They explain how they made their estimate and comment on the amount of error in their estimate.

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Standard 5: ALGEBRA, FUNCTIONS, AND PATTERNS

Students use algebraic concepts, functions, patterns, and relationships to solve problems.

Benchmarks

12.5.1 Use algebraic procedures to manipulate mathematical expressions.
12.5.2 Solve equations, inequalities, and systems.
12.5.3 Represent and describe relations algebraically, numerically, and graphically.
12.5.4 Create, manipulate, and apply matrices to real life situations.
12.5.5 Develop and analyze a variety of algorithms.
12.5.6 Understand and apply the process of recursion.
12.5.7 Use patterns and functions to model problems.
12.5.8 * Understand the ideas of convergence, limit of functions, and infinite series.

Examples of Specific Knowledge that Support the Standard and Benchmarks

12.5.1 rationalize, distributive property, associative property, commutative property
12.5.2 roots, addition and multiplication properties of equality and inequality, substitution, linear combination, graphing, matrices
12.5.3 functions (linear, quadratic, cubic), function notation, sine, cosine, tangent, yintercept, maximum and minimum value, continuity, asymptotes, transformations, circle, hyperbola, ellipse, *limits of a function, functions (*rational, *radical, exponential, *logarithmic, *trigonometric, and *inverse)
12.5.4 matrix, *determinant, *inverse matrix
12.5.5 logic, induction, deduction, geometric constructions, greatest common factor, least common multiple
12.5.6 Fibonacci sequence, arithmetic sequence, geometric sequence, *self-similarity, *fractals, *chaos, *iterations
12.5.7 linear programming, discrete situations
12.5.8 *limit of a series, *convergence, *divergence

Examples of Activities that Support the Standard and Benchmarks

12.5.1 The National Weather Service uses the following formula to compute wind chills:
WC = .0817(3.71Öv + 5.81-.25v)(T-91.4)+91.4.
In this formula, WC is the wind chill, V is the wind speed in mph, and T is the temperature in degrees Fahrenheit. Students verify the formula by entering local weather information. Students solve the formula for T and V.

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Examples of Activities that Support the Standard and Benchmarks

12.5.2 Students use linear programming to solve optimization problems using systems of linear inequalities. For example, a contractor builds two types of homes. The first type requires one lot, $12,000 capital and 150 man-days of labor to build and sells for a profit of $2400. The second type of home requires one lot, $32,000 capital and 200 man-days to build and is sold for a profit of $3400. The contractor owns 150 lots, has available $2,880,000 capital, and is able to provide 24,000 man-days of labor. How many homes of each type should she build in order to realize the greatest profit?

12.5.3 Students play a game of "Green Globs" using only one function type (such as parabolas or lines). Students hand in a listing of the equations used and a printout of the final graph. ("Green Globs" is a computer program available from Sunburst Communications.)

12.5.3 Using appropriate technology, students explore the transformations of various functions caused by changes in parameters. Based on these explorations, students present written accounts of general rules for transformations of functions' graphs.

12.5.3 Students work in pairs with one student plotting a polynomial function on a graphing calculator and the other student finding an appropriate equation that would fit the curve by observing the roots, relative extrema, and end behavior of the graph.

12.5.3 Given three coordinates and the assumption that a projectile follows a parabolic path, students find the quadratic equation describing the projectile's position. This involves solving a system of three equations in three unknowns.

12.5.4 Probability matrices and repeated multiplication can be used to investigate Markov processes such as likelihood of rain over time or distribution of rental cars based on initial conditions and empirical probability information. Given a description of such situations, students develop the use of matrices and matrix multiplication to represent the probability computations in a more compact form and investigate the convergence or stability of the process.

12.5.4 Students use matrices to explore transformations, such as reflections and translations, of geometric figures in the coordinate plane.

12.5.5 Students describe the outcome of the following program:

```
10 LET A = 0
20 FOR I = 1 TO 5
30 A = A + 2I
40 PRINT A
50 NEXT I
60 END
```

12.5.6 Students construct the Sierpinski Triangle (Gasket). They start with an arbitrary solid triangle and then remove the triangle whose vertices are the midpoints of the sides of the triangle. For each of the solid triangles in the previous step, they repeat the removal procedure. They continue the process to establish the Sierpinski Triangle.

12.5.7 Students study the (inverse square) relationship between distance from source and light intensity using appropriate technologies to generate data and then model the relationship.

12.5.8 Given the following information, students solve the problem. A wolf population in a certain region grows according to a logistics growth function:

$$w(t) = 5000/\{1+e^{-.5t}\}$$

where t is the time in years and w(t) is the number of wolves present at that time. Find the limit that the wolf population approaches by constructing a table and by graphing, using appropriate technologies.

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Summary of Benchmarks By Standard

Standard 1: NUMBER AND OPERATION

Students understand and use basic and advanced concepts of number and number systems.

Grades K-4 Benchmarks	
4.1.1	Construct and interpret number meanings through real-world experiences.
4.1.2	Understand the characteristics and properties of our numeration system.
4.1.3	Understand how arithmetic operations are related to one another in addition, subtraction, multiplication, and division.
4.1.4	Rename, order, and compare numbers.
4.1.5	Know and use basic facts and computational algorithms for whole numbers, fractions and decimals.
4.1.6	Use estimation strategies in working with quantities, measurement, computation, and problem solving.
4.1.7	Understand and communicate strategies to solve a wide variety of problems.

Grades 5-8 Benchmarks	
8.1.1	Understand numbers, number systems and ways of representing numbers.
8.1.2	Apply number theory concepts in mathematical problems.
8.1.3	Determine appropriate information and methods to solve problems.
8.1.4	Compute with real numbers using appropriate computational methods for given situations.
8.1.5	Apply appropriate estimation strategies to determine if a solution is reasonable.

Grades 9-12 Benchmarks	
12.1.1	Know and use the real number system, its subsets and properties.
12.1.2	Identify complex numbers and understand their relevance in solving equations.
12.1.3	Use basic set relations and operations with appropriate notation.
12.1.4	Understand the meaning of operations and how they relate to each other.
12.1.5	Apply advanced estimation skills and appropriate computational methods to attain reasonable solutions.
12.1.6	Understand the properties and basic theorems of roots, exponents, and logarithms.

Standard 2: GEOMETRY AND SPATIAL SENSE

Students understand and apply geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations.

Grades K-4 Benchmarks	
4.2.1	Know the characteristics of two- and three-dimensional shapes.
4.2.2	Understand how two- and three-dimensional shapes can be changed by combining and dividing.
4.2.3	Understand that geometry is found within and outside mathematics.

** Indicates the word or phrase is defined in the Glossary*

Grades 5-8 Benchmarks

8.2.1 Understand the relationships between two- and three-dimensional models using a variety of materials and tools.
8.2.2 Know, and reason informally about, properties of two- and three-dimensional figures.
8.2.3 Know the components of the coordinate plane.
8.2.4 Determine perimeter, circumference, and area in two dimensions, and surface area and volume in three dimensions.
8.2.5 Use transformations and symmetry to investigate similar figures.

Grades 9-12 Benchmarks

12.2.1 Understand and apply the properties of two- and three-dimensional figures.
12.2.2 Construct basic geometric figures using appropriate tools.
12.2.3 Understand the concepts of congruence, similarity, and symmetry.
12.2.4 Apply transformations to basic shapes.
12.2.5 Apply the Pythagorean Theorem to solve problems.
12.2.6 Apply basic trigonometric ratios to solve real-world problems.
12.2.7 Apply measurements and formulas in computations of perimeter, area, and volume.
12.2.8 Generate geometric conjectures inductively and validate them deductively.
12.2.9 Apply geometric properties to a coordinate system.

Standard 3: DATA ANALYSIS, STATISTICS, AND PROBABILITY

Students use data collection and analysis techniques, statistical methods, and probability to solve problems.

Grades K-4 Benchmarks

4.3.1 Collect, organize and display data.
4.3.2 Formulate and solve problems that involve data.
4.3.3 Draw conclusions based on probability.
4.3.4 Use technology and materials as tools to display data.

Grades 5-8 Benchmarks

8.3.1 Collect, read, and display data using appropriate techniques and technology.
8.3.2 Display and use measures of central tendency and measures of variability.
8.3.3 Evaluate arguments that are based on statistical claims.
8.3.4 Identify basic trends in tables and graphs and use these trends to make predictions.
8.3.5 Determine probabilities through experiments or simulations.
8.3.6 Understand and apply the basic notions of probability.
8.3.7 Use counting strategies to determine all the possible outcomes.

Grades 9-12 Benchmarks

12.3.1 Design, implement, and present statistical studies.
12.3.2 Sample data and understand the role of sampling in data analysis.
12.3.3 Use counting strategies.
12.3.4 Calculate theoretical and experimental probabilities.
12.3.5 Calculate and interpret measures of central tendency and variance.
12.3.6 Use regression techniques to determine and interpret the curve of best fit.
12.3.7 Draw inferences and predict outcomes from data expressed in a variety of ways.

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Standard 4: MEASUREMENT

Students use concepts and tools of measurement to describe and quantify the world.

Grades K-4 Benchmarks	
4.4.1	Select and use the appropriate tool to determine measurements of length, area, perimeter, volume, and angle size.
4.4.2	Use estimation strategies in working with quantities, measurement, computation, and problem solving.
4.4.3	Apply a variety of techniques, tools, and formulas to determine measurements.
4.4.4	Know and use units of time, money, and temperature.

Grades 5-8 Benchmarks	
8.4.1	Select appropriate units and scale to estimate and measure.
8.4.2	Select and use appropriate measurement unit and tools to solve problems.
8.4.3	Use formulas and procedures to solve problems involving measurement.

Grades 9-12 Benchmarks	
12.4.1	Understand attributes, units, and systems of measurement.
12.4.2	Apply a variety of techniques, tools, and formulas to determine measurements.
12.4.3	Measure physical quantities and determine measurement error.
12.4.4	Use estimation in the measurement process.

Standard 5: ALGEBRA, FUNCTIONS, AND PATTERNS

Students use algebraic concepts, functions, patterns, and relationships to solve problems.

Grades K-4 Benchmarks	
4.5.1	Understand when a simple pattern exists, identify the rule that generates the pattern, and use that information to solve problems.
4.5.2	Represent and describe mathematical relationships using symbols for variables.
4.5.3	Solve problems with unknown variables.
4.5.4	Use basic counting strategies to determine all possible outcomes.

Grades 5-8 Benchmarks	
8.5.1	Find, represent, describe, and analyze patterns, functions, and relations using tables, graphs, verbal descriptions, and standard algebraic notation.
8.5.2	Understand the concepts of functions, variables, expressions, equations, and inequalities.
8.5.3	Solve linear equations, inequalities, and systems of equations in problem-solving situations using a variety of methods and a variety of tools.

Grades 9-12 Benchmarks	
12.5.1	Use algebraic procedures to manipulate mathematical expressions.
12.5.2	Solve equations, inequalities, and systems.
12.5.3	Represent and describe relations algebraically, numerically, and graphically.
12.5.4	Create, manipulate, and apply matrices to real-life situations.
12.5.5	Develop and analyze a variety of algorithms.
12.5.6	Understand and apply the process of recursion.
12.5.7	Use patterns and functions to model problems.
12.5.8	* Understand the basic ideas about convergence, limit of functions, and infinite series.

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Glossary

Absolute error - The difference between the true value and estimated value.

Algebra - The branch of mathematics that is the generalization of the ideas of arithmetic.

Algebraic methods - The use of symbols to represent numbers and signs to represent their relationships.

Algorithm - A step-by-step procedure.

Apothem - The perpendicular distance from the center to a side in a regular polygon.

Box plot (also called a **box-and-whiskers plot**) - A graphic method for showing a summary of data using median, quartiles, and extremes of data. A box plot makes it easy to see where the data are spread out and where they are concentrated. The longer the box, the more the data are spread out.

Combinations - Subsets chosen from a larger set of objects in which the order of the items doesn't matter (for example, the number of different committees of three that can be chosen from a group of twelve members).

Complex numbers - Numbers that can be written in the form a + bi, for example, -2.7 + 8.9i, where a and b are real numbers and i = •-1.

Congruent (or the **concept of congruence**) - Two figures are said to be congruent if they are the same size and shape.

Coordinate geometry - Geometry based on the coordinate system.

Coordinate system (also called **rectangular coordinate system**) - A method of locating points in the plane or in space by means of numbers. A point in a plane can be located by its distances from both a horizontal and a vertical line called the axes. The horizontal line is called the x-axis. The vertical line is called the y-axis. The pairs of numbers are called ordered pairs. The first number, called the x-coordinate, designates the distance along the horizontal axis. The second number, called the y-coordinate, designates the distance along the vertical axis. The point at which the two axes intersect has the coordinates (0,0) and is called the origin.

Conic sections - Cross sections of a double napped cone that produce either a circle, parabola, ellipse, or hyperbola.

Conjecture - A statement that is to be shown true or false. A conjecture is usually developed by examining several specific situations.

Convergence - The process of approaching the limit of a function.

Deductive reasoning - Applying a general rule to a specific example in order to arrive at a logical conclusion.

Descriptive statistics – A branch of statistics that consists of the collection, organization, summarization, and presentation of data.
Dilation - A transformation that either enlarges or reduces a geometric figure proportionally.

** Indicates the word or phrase is defined in the Glossary*

Direct variation - A linear function of the form $y = cx$, where c is the constant of variation and c • 0. In this function, y is directly proportional to x, that is, y varies directly as x varies.

Discrete mathematics - The study of mathematical properties of sets and systems of mathematics that have a countable number of elements.

Discrete structures - Some examples of discrete structures are finite graphs, networks, sequences, recurrence relations, and voting methods.

Experimental (empirical) probability - A probability based on the results of an experiment or series of trials.

Exponential function - A function that has an equation of the form $y= ax_n$. These functions are used to study population growth or decline, radioactive decay, and compound interest.

Factorial - The product of all the integers from 1 up to the integer in question. For example, 4! = 1 x 2 x 3 x 4 = 24.

Fermi problems - Problems in which an estimated solution is arrived at by computation using approximate values. An example of a Fermi problem is calculating the number of basketballs that would fit in a classroom.

Fibonacci sequence - The first two numbers of the Fibonacci sequence are 1; every other number is the sum of the two numbers that immediately precede it. Therefore, the first 14 numbers in the Fibonacci sequence are: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377.

Fractal - A geometric shape that is self-similar and has fractional dimensions. Natural phenomena such as the formation of snowflakes, clouds, mountain ranges, and landscapes involve patterns. Their pictorial representations are fractals and are usually generated by computers.

Function - A correspondence between two sets which assigns to each element of the first set (the domain) a unique element in the second set (the range).

Geometry - A branch of mathematics that deals with the measurement, properties, and relationships of points, lines, angles, and two- and three-dimensional figures.

Glyph - A simple picture or figure whose parts represent information about a given subject.

Inductive reasoning - Arriving at a conjecture through the use of patterns where a particular situation is applied to a general situation.

Inferential statistics - A branch of statistics that consists of generalizing from samples to populations, performing hypothesis testing, determining relationships among variables, and making predictions

Integers - The set of numbers consisting of the counting numbers (that is, 1, 2, 3, 4, 5, ...), their opposites (that is, negative numbers, -1, -2, -3, ...), and zero.

** Indicates the word or phrase is defined in the Glossary*

Irrational numbers - The set of numbers, which cannot be represented as fractions. Examples are •2, the cube root of 29, e , and •.

Inverse variation - A variation stated in the form $y = k/x$, where k is the constant of variation. In this equation the values of y get smaller as the values of x get bigger.

Isometry - A transformation that preserves size and shape.

Limit of a function - The value of y as x approaches a given point or infinity.

Linear function - A function that has a constant rate of change.

Logarithm - Alternate way to express an exponent. For example, $\log_2 8=3$ is equivalent to 2 to the third power = 8.

Manipulatives - A tool used to help visualize and solve problems such as: Base 10 blocks, Unifix cubes, linking cubes, etc.

Matrix (pl. matrices) - A rectangular array of numbers (or letters) arranged in rows and columns.

Measures of central tendency - Numbers which in some sense communicate the "center" or "middle" of a set of data. The mean, median, and mode of statistical data are all measures of central tendency.

Mira - A colored plexiglass drawing device that uses reflection to aid in geometrical constructions

Model - To make or construct a physical or mathematical representation.

Number sense - An understanding of number. This would include number meanings, number relationships, number size, and the relative effect of operations on numbers.

Number theory - The study of the properties of the natural numbers.

Optimization problems - Real-world problems in which, given a number of constraints, the best solution is determined. For example, finding the best number of nonstop flights from Denver to San Francisco given the cost of fuel, number of passengers, number of crew required, etc.

Pascal's triangle - A triangular array of numbers in which each number is equal to the sum of the two numbers above it. The n+1 row of Pascal's triangle gives the coefficients of the expansion of $(a + b)^n$.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

** Indicates the word or phrase is defined in the Glossary*

Patterns - Regularities in situations such as those in nature, events, shapes, designs, and sets of numbers (for example, spirals on pineapples, geometric designs in quilts, the number sequence 3,6,9,12,...).

Permutations - All possible arrangements of a given number of items in which the order of the items makes a difference (for example, the different ways that a set of four books can be placed on a shelf).

Polar coordinate system - A system in which any point in a plane can be identified by its distance from the origin and its angle inclination. It is an alternative to the rectangular coordinate system.

Polygon – A simple enclosed figure made up of line segments.

Probability - The likeliness or chance of an event occurring.

Problem-solving situations - Contexts in which problems are presented that apply mathematics to practical situations in the real world, or problems that arise from the investigation of mathematical ideas.

Proportion - The equality of two ratios.

Pythagorean Theorem - In a right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse ($a^2 + b^2 = c^2$).

Quadratic function - A function that has an equation of the form $y = ax^2+bx+c$, where $a \neq 0$. These functions are used to describe the flight of a ball and the stream of water from a fountain.

Rational numbers - A number that can be expressed in the form a/b, where a and b are integers and $b \neq 0$, for example, 3/4, 2/1, or 11/3. Every integer is a rational number, since it can be expressed in the form a/b, for example, 5 = 5/1. Rational numbers may be expressed as fractional or decimal numbers, for example, 3/4 or .75. Finite decimals, repeating decimals, and mixed numbers all represent rational numbers.

Real numbers - All rational and irrational numbers.

Real-world problems (also called **real-world experiences**) - Quantitative problems that arise from a wide variety of human experiences which may take into consideration contributions from various cultures (for example, Mayan or American pioneers), problems from abstract mathematics, and applications to various careers (for example, making change or calculating the sale price of an item).

Recursion - The process of applying an algebraic formula to a previously acquired number in order to obtain a sequence of numbers.

Reflection (also called a **flip**) - A transformation which produces the mirror image of a geometric figure.

Regression - Statistical technique that predicts the equation that best fits the data.

Relative error - The ratio between the absolute error and the true value.

Rotation (also called a **turn**) - A transformation which turns a figure about a point a given

number of degrees.

Scatter plots (also called **scatter diagram** or **scattergram**) - A graph of the points representing a collection of data.

Series – An indicated sum of the terms of a sequence.

Sequence – A set of numbers, called terms arranged in a particular order; a function whose domain is the set of positive integers.

Similarity - Objects or figures that are the same shape are similar figures. They are not necessarily the same size. If two figures are similar, we say that there is similarity between the figures.

Spatial visualization (also called **spatial reasoning**) - A type of reasoning in which a person can draw upon his or her understanding of relationships in space, the three-dimensional world. For example, spatial reasoning is demonstrated by one's ability to build a three-dimensional model of a building shown in a picture. A person who uses spatial visualization is said to have spatial sense.

Square root - That number which when multiplied by itself produces the given number. For example, 5 is the square root of 25, because 5 x 5 = 25.

Statistics - The branch of mathematics which is the study of the methods of collecting and analyzing data. The data are collected on samples from various populations of people, animals, or products. Statistics are used in many fields, such as biology, education, physics, psychology, and sociology.

Stem-and-leaf plot - A frequency distribution made by arranging data. It is one way of visually portraying data that is frequently used in newspapers and magazines because it provides an efficient way of showing information as well as comparing different sets of data.

Symmetry - The correspondence in size, form, and arrangement of parts on opposite sides of a plane, line, or point. For example, a figure that has line symmetry has two halves, which coincide if folded along its line of symmetry.

Systems (also called **simultaneous equations**) - Two or more equations in two or more variables considered together or simultaneously.

Theoretical probability - Probability based on the prior knowledge of the characteristics of the experiment.

Transcendental – a real number which is not an algebraic number such as π, e, and $2^{\sqrt{3}}$.

Transformation - The process of changing one configuration or expression into another in accordance with a rule. Common geometric transformations include translations (slides), rotations (turns), and reflections (flips).

Translation (also called a **slide**) - A transformation that moves a geometric figure by sliding. Each of the points of the geometric figure moves the same distance in the same direction.
Trigonometric ratios - The ratios of the lengths of pairs of sides in a right triangle. There are three basic trigonometric ratios used in trigonometry: sine (sin), cosine (cos), and tangent (tan).

** Indicates the word or phrase is defined in the Glossary*

Tree diagram - An illustration of all the possible results for a process with several stages. An example would be listing all the combinations of options available when buying a new car.

Trigonometry - A branch of mathematics that combines arithmetic, algebra, and geometry. Trigonometry is used in surveying, navigation, and various sciences such as physics.

Variable - A quantity that may assume any one of a set of values. In the equation $2x + y = 9$, x and y are variables.

Vector - A quantity which has both magnitude and direction. Vectors may be interpreted as physical quantities such as velocity and force.

Venn diagram - A picture that illustrates relationships between sets.

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